Old Lotto System

Let us use a lotto once popular (now largely not used in the US) to illustrate the relationship between the counting theorem and the rules of permutation and combination.

Five winning numbers are drawn (without replaement) from numbers 1 through 90. To win the ultimate jackpot a player would have to have the five numbers on the ticket, but in any order. So if the winning numbers are 24 - 25 - 35 - 40 - 89, tickets 89 - 40 - 24 - 25 - 35 and 35 - 24 - 40 - 89 - 25, etc would be winners. So even though the order of the numbers is not important to the player, it is relevant from the perspective of the Lotto Company in figuring out the probability of wins and potential payouts.

Let us use the counting theorem to compute the number of possible outcomes

Event 1	Event 2	Event 3	Event 4	Event 5
$\boxed{\text{90 ways}}$	$\boxed{\text{89 ways}}$	$\boxed{\text{88 ways}}$	$\boxed{\text{87 ways}}$	$\boxed{\text{86 ways}}$

So number of possible outcomes $= 90.89.88.87.86$

$= 5,273,912,160$

Alternately we could have used the rule of permutation because a different order of the numbers on the ticket will still win.

$${}_{90}P_5 = \frac{90!}{(90-5)!} = \frac{90.89.88.\dots.1}{85.84.85\dots1} = 90.89.88.87.86.$$

$$= 5,273,912,160$$

So if five numbers will be drawn, the number of possible winning permutations would be

$${}_5P_5 = \frac{5!}{(5-5)!} = 5.4.3.2.1 = 120$$

The probability of winning this lotto's ultimate prize is $\frac{120}{5,273,912,160}$ or odds of approx. 43,949,268 to 1

Madeup Lottery

Numbers 1 through 90

Five ~~numbers~~ winning numbers drawn without replacement

$\boxed{90}$ $\boxed{89}$ $\boxed{88}$ $\boxed{87}$ $\boxed{86}$

Number of possible winning tickets

$$(90)(89)(88)(87)(86) = 5{,}723{,}912{,}160$$

If we express a (one) players chance of winning as

___ : 1 or 1 in ___ we call this the

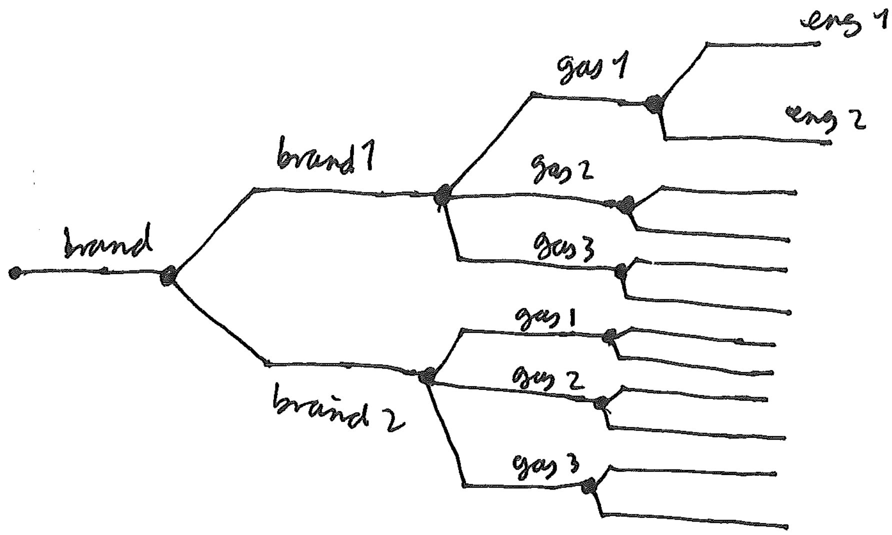
'odds' of winning.

Events : brand, gas mile, engine

2 3 2

~~(3)(2)(3)~~

$(2)(3)(2) = 12$



Combination

back to Madeup Lottery. From the perspective of the player order is not relevant. So let us apply combination.

The odds of winning with any 2 numbers:

$$_{90}C_2 = \frac{90!}{(90-2)!\,2!} =$$

Three numbers

$$_{90}C_3 = \frac{90!}{(90-3)!\,3!} =$$

Four numbers

$$_{90}C_4 = \frac{90!}{(90-4)!\,4!}$$

Five numbers: $_{90}C_5 = \dfrac{90!}{(90-5)!\,5!} = 43{,}949{,}268$

which matches the odds from the previous computation based on permutation.

So all the counting rules we have seen so far involve counting WITHOUT replacement.

Also, all the rules are derived from the fundamental counting them.

In working counting problems (also called Combinatorics) it is important you figure out the law applicable to your problem and then apply it.

Problem 3.19 Combination

a) $_6C_2 \cdot {}_4C_2$

No ways of men = 15

No ways of women = 6

No of way total = 90

b) No ways of women $= \cancel{(_6C_2)\left[(_3C_2) + {}_3C_2\right]}$

$= {}_6C_2\left[{}_2C_2 + 2 + 2\right]$

$= 15(5) = 75$

~~Problem 3.31 Probability~~

$\cancel{P(2cc \quad P(cc) = 18/30 \quad P(ms) = 12/30}$

$P(2cc \text{ and } 2ms) = \cancel{2(18/30 \cdot 2 \cdot 12/30)} = \cancel{\frac{18}{30} \cdot \frac{17}{30} \cdot \frac{12}{3}}$

$\cancel{_{18}C_2 \cdot {}_{12}C_2 = \frac{1}{143.66}}$

Problem 3.29

a)

total possible outcomes = (6)(6) = 36

$\{6,1\}$ $\{5,2\}$ $\{4,3\}$

total ways to get 7 : $\{1,6\}, \{2,5\}, \{3,4\} = 6$

~~Note: order does not matter we need total of 7.~~

$P(7) = 6/36 = \cancel{0.083}\ 1/6$

b) $\{5,6\}, \{6,5\}$

$P(11) = 2/36 = \cancel{0.0278}\ 1/18$

c) $P(7 \text{ or } 11) = \overset{1/6}{\cancel{0.083}} + \overset{1/18}{\cancel{0.0278}} = \cancel{0.111}\ \frac{3+1}{18} = \frac{4}{18} = \frac{2}{9}$

d) $\{2,1\}, \{1,2\}$

$P(3) = 2/36 = \cancel{0.0278}\ 1/18$

e) $P(2) = 1/36\ \cancel{= 0.0278}$

$P(12) = 1/36\ \cancel{= 0.0278}$ $\{6,6\}$

$P(2 \text{ or } 12) = 0.0278 + 0.0278 = 0.0556$ or $1/18$

f) $P(2, 3, \text{or } 12) = 0.0278 + 0.0278 + 0.0556 = 0.111$ or $\frac{1}{9}$

Problem 3.31

~~No of ways of selecting = 18.12 = 216~~

No of ways of selecting 2 cc from 18

$= {}_{18}C_2 = 153$

No of way of selecting 2 ms from 12

$= {}_{12}C_2 = 66$

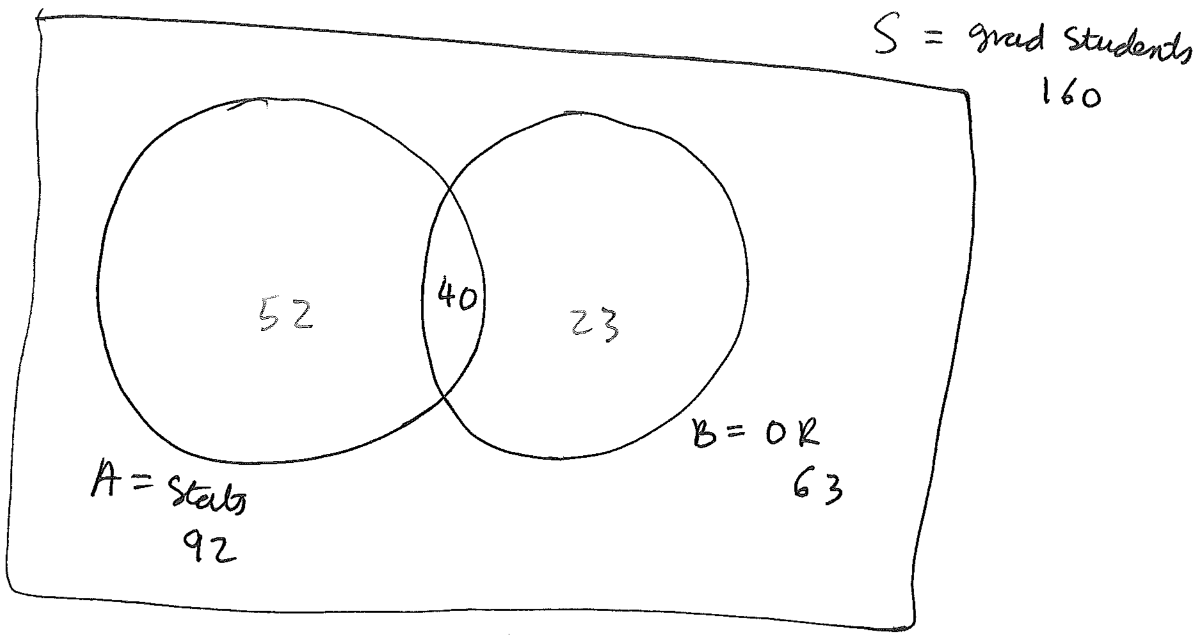
No of ways of selecting 4 cars from 30

$= {}_{30}C_4 = 27405$

$$P(2cc \text{ AND } 2ms) = \frac{(153)(66)}{27405} = 0.3684$$

Problem 3.33

Problem 3.33



$92 + 63 = 155$?? uh oh!

$92 + 63 + 40 = 195$ yikes!! I'm screwed!

So the numbers for A and B include those doing both. We must isolate the 'double dipping'.

A only $= 92 - 40 = 52$

B only $= 63 - 40 = 23$

So now $A \cup B = 52 + 40 + 23 = 115$

So not doing any $\overline{A \cup B} = 160 - 115 = 45$

Problem 3.47

$P(\text{design}) = 0.16 \qquad P(\text{materials}) = 0.24$

$P(\text{design } \underline{\text{AND}} \text{ materials}) = 0.11$

$P(\text{at least something}) = P(\text{nothing}^c)$

Visualize a Venn Diagram !! like in Prob 3.33

$P(\text{d only}) = 0.16 - 0.11 = 0.05$

$P(\text{m only}) = 0.24 - 0.11 = 0.13$

$P(\text{Something}) = 0.05 + 0.13 + 0.11 = 0.29$

$$P(\text{only } 1) = P(\text{d only}) + P(\text{m only})$$
$$= 0.05 + 0.13$$
$$= 0.18$$

Problem 3.65

$P(SE) = 45/60$ $\quad P([illegible]) = 15/60$

$P(uPT) = 2/60$

$\frac{45}{60}\cdot\frac{44}{59}$ $\quad \frac{15}{60}\cdot\frac{14}{59} = \frac{14}{236} = \frac{7}{118}$

$P(\text{2 parts unloaded in PT}) = \frac{2}{60}\cdot\frac{2}{60}$

$P(SE/uPT) = \frac{P(SE \cap uPT)}{P(uPT)}$

$\frac{45}{60}\cdot\frac{44}{59} = 0.559 \approx \frac{33}{59}$

$\frac{15}{60}\cdot\frac{14}{59} = 0.059322 \approx 7/118$

$\frac{45}{60}\cdot\frac{15}{59} = 0.19067$

3.97

$P(Ex|SE) = 0.125$

$P(Ex|ME) = 0.20$

$P(Ex|OF) = 0.40$

$P(Ex|PA) = 0.75$

Using Bayes Theorem.

$P(SE|Ex) = \frac{\overset{\text{prior}}{0.3}(0.125)}{0.30(0.125) + 0.4(0.2) + 0.15(0.4) + 0.15(0.75)}$

$= \frac{0.3(0.125)}{0.3275} = 0.229$

$P(ME|Ex) = \frac{0.4(0.2)}{0.3275} = 0.244$

$P(OF|Ex) = \frac{0.15(0.4)}{0.3275} = 0.183$

$P(PA|Ex) = \frac{0.15(0.75)}{0.3275} = \boxed{0.343}$

3.98

$P(\text{3 days}) = 0.7$

$P(\text{2, 3 days}) = 0.7 \cdot 0.7 = 0.49$

Problem 3.79

$P(\text{not Det} | \text{Pres}) = \frac{\quad}{P(\text{Pres})}$

~~$P(c/p) = 0.7$~~ $P(\text{correct}) = 0.7$

~~$P(c/NP) = 0.2$~~ $P(\text{false}) = 0.2$

$P(c) = 0.1$

$P(\text{Det}) = P(\text{Pres}).P(\text{Det}|\text{Pres}) + P(\text{not Pres}).P(\text{Det}|\text{not Pres})$

$P(\text{Det}|\text{Pres}) = \frac{P(\text{Det} \cap \text{Pres})}{P(\text{Pres})}$

$P(c/$

b) $P(\text{Pres}/\text{not Det}) = \frac{P(\text{Pres} \cap \text{not Det})}{P(\text{not Det})} = \frac{P(\text{not Det}/\text{Pres})P(\text{Pres})}{P(\text{not Det})}$

$= \frac{(1-0.7)(0.1)}{1-0.28} = 0.04$

a). $P(\text{Det}/\text{Present}) = 0.7$

$\frac{45}{118}$ $P(\text{Det}/\text{not Present}) = 0.2$

$P(\text{Present}/\text{Det}) = \frac{P(\text{Pres} \cap \text{Det})}{P(\text{Det})} = \frac{P(\text{Det}/\text{Pres})\,P(\text{Pres})}{P(\text{Det})}$

$= \frac{0.7 * 0.1}{\cancel{0.7+0.2}} = \cancel{0.07778}$

$0.1(0.7) + 0.9(0.2) = 0.28$

3.69

a) $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{256}$

b) $\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \left(\frac{1}{6} + \frac{1}{6}\right) = \frac{1}{648}$

c) ~~$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$~~ = No ways $= 3.3.3.3.3 = 3^5 = 243$

3·92

~~P~~ Events are :

satelitte explodes during launch $= A$
guidance system failure $= B$

$P(A) = 0.0002 \qquad P(B) = 0.0005$

a) $P(A^c) = 1 - P(A) = 1 - 0.0002 = 0.9998$

b) $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$
$= 0.0002 + 0.0005$
$= 0.0007$

c) $P(A^c \text{ or } B^c) = P(A^c \cup B^c) = P(A^c) + P(B^c)$
$= [1 - P(A)] + [1 - P(B)]$
$= 0.4998 + 0.9995$
$= 1.9993$ WHAT!!!

So obviously this approach fails. We need to find complement of what we did in part b).

So neither A nor B is complement of A or B.

$P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - 0.0007$
$= 0.9993$

This is therefore the probability of a successful launch.

Conditional Probability

3.79

$P(\text{corrosion}) = 0.1$

$P(\text{present}/\text{corrosion}) = 0.7$

$P(\text{present}/\text{no corrosion}) = 0.2$

a) ~~we want~~
Let us convert to a more 'user friendl
notation

present = A meaning test indicates corro
is present

corrosion = B meaning there is corrosio
in the pipe.

$P(B) = 0.1 \Rightarrow P(B^c) = 0.9$

$P(A/B) = 0.7$

$P(A/B^c) = 0.2$

a) $P(B/A) = \dfrac{P(B \cap A)}{P(A)}$ —(1)

First find $P(B \cap A)$

since $P(B \cap A) = P(A \cap B)$

we could have written the Conditional probability law as

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A \cap B) = P(B \cap A) = P(A/B)\,P(B)$$

$$= (0.7)(0.1)$$
$$= 0.07$$

Now find $P(A)$

so test indicates presence of corrosion under two cases, i) it is present,
ii) it is not present

So we can use Law of Total Probability to get $P(A)$.

$$P(A) = \sum P(B_i) . P(A/B_i)$$

$$= P(\text{corrosion}) . P(\text{present}/\text{corrosion})$$
$$+ P(\text{no corrosion}) . P(\text{present}/\text{no corrosion})$$

or

$$= P(B) . P(A/B) + P(B^c) . P(A/B^c)$$

$= (0.1)(0.7) + (0.9)(0.2)$

$= 0.25$

Now plugging in Eqn (1)

$P(B/A) = P(\text{corrosion}/\text{present})$

$= \dfrac{0.07}{0.25}$

$= 0.28$

b) $P(\text{corrosion}/\text{not present}) = P(B/A^c)$

Based on this equipment we can have corrosion and the device says it is present, and we can have corrosion with the device saying it is not present. So we can use Law of Total Probability

$P(\text{corrosion}) = P(\text{present}) \cdot P(\text{corrosion}/\text{present})$

~~+ P(not present/~~

$+ P(\text{not present}) \cdot P(\text{corrosion}/\text{not present})$

$= P(A) \cdot P(B/A) + P(A^c) \cdot P(B/A^c)$

so plugging what we have and solve for the unknow term

$$0.1 = 0.25(0.28) + (1-0.25) \cdot P(B/A^c)$$

$$P(B/A^c) = 0.04$$

This is the probability that there ~~test~~ is corrosion but the test says otherwise.

Students, rework this problem using Bayes' Theorem!

3.79 Using Bayes' Theorem

$$P(B/A) = \frac{P(B) \cdot P(A/B)}{P(B) \cdot P(A/B) + P(B^c) \cdot P(A/B^c)}$$

$$= \frac{0.1\,(0.7)}{0.1\,(0.7) + 0.9\,(0.2)}$$

$$= 0.28$$

What the .... !!!
So 3 steps versus 3 pages using Conditional Probability plus Total Probability. Sorry, as your instructor I had to show you everything.

However to use Bayes' Theorem you must be sure you understand the ~~ex~~ experiment and know what you are doing, else you will get it wrong and there will be little opportunity to pin-point where you got it wrong.

From my professional work Bayes is used when you have several — 100s, 1000s of conditions. Otherwise for 3, 4 conditions we typically use Conditional Probability law.